

## PROJECT ADMINISTRATION DATA SHEET



ORIGINAL



REVISION NO. \_\_\_\_\_

Project No. E-24-659 (R5834-1A0)GTRC/~~GTR~~DATE 10/28/86Project Director: R. F. SerfozoSchool ~~XXX~~ ISyESponsor: Air Force Office of Scientific ResearchBolling AFB, DC 20332-6448Type Agreement: Grant No. AFOSR-84-0367 Amendment BAward Period: From 9/30/86 To 9/30/87 (Performance) 11/30/87 (Reports)

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## ADMINISTRATIVE DATA

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1) Sponsor Technical Contact:

2) Sponsor Admin/Contractual Matters:

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## RESTRICTIONS

See Attached N/A Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval — Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of \$500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT if acquired at a cost of less than \$5,000.

## COMMENTS:

\*Aggregate total of award and all amendments to-date (Basic & Amend. A)Continuation of project E-24-628

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEETDate 5/11/88Project No. E-24-659School/Lab ISYEIncludes Subproject No.(s) N/AProject Director(s) R. F. SerfozoGTRC/~~XXX~~Sponsor Air Force Office of Scientific ResearchTitle Stochastic Flows in NetworksEffective Completion Date: 9/30/87 (Performance) 11/30/87 (Reports)

## Grant/Contract Closeout Actions Remaining:

☐

None

☒

Final Invoice or Copy of Last Invoice Serving as Final

☒

Release and Assignment

☒

Final Report of Inventions and/or Subcontract:

Patent and Subcontract Questionnaire  
sent to Project Director ☒☒

Govt. Property Inventory &amp; Related Certificate

☐

Classified Material Certificate

☐

Other \_\_\_\_\_

Continues Project No. E-24-628

Continued by Project No. \_\_\_\_\_

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# ANNUAL TECHNICAL REPORT

Project Title: Stochastic Flows in Networks

Grant No.: AFOSR 84-0367

Project Director: Professor Richard F. Serfozo  
Industrial and Systems Engineering  
Georgia Institute of Technology

December 1987

## Stochastic Flows in Networks

This report is a brief summary of our reserach progress in the one year from October 1, 1986 to September 30, 1987. We worked on several research themes. The following is an overview of our results.

### 1. **Handbook on Point Processes**

Our work on this will appear in:

Serfozo, R.F. (1989). Point Processes. A chapter in the ORSA Handbook on Stochastic Processes. (The table of contents appears in Appendix A1.)

The Operations Research Society of America is preparing a handbook on stochastic processes, comparable to the handbooks on statistics and economics. We wrote a chapter on point processes covering most of the fundamental results that have appeared in the literature. Because this subject is still evolving, our work required more effort than an established topic. We brought together many scattered results, refined many proofs, gave new insights on applications and presented a few new results. For instance, we showed that the key renewal theorem has a wider range of applications than that suggested in the literature. In doing so, we gave a new result on the rate of convergence of certain functionals of Markov processes to their limits. We may develop these ideas further with a view to writing a piece on new insights on the key renewal theorem.

## 2. Poisson Functionals of Stochastic Processes

Our work on this has been documented in:

Serfozo, R.F. (1987). Poisson functionals of Markov processes and queueing networks. Technical report. (The abstract appears in Appendix A2).

Many point processes arise as functionals of stochastic processes. For instance, in a Markovian network of queues, one might be concerned with the point process of times at which customers move between two sectors of the network. And, if there is synchronous movements of customers, one might also be interested in the numbers of customers that move between the two sectors (these numbers being the "marks" of the respective times of the movements). One can formulate such a point process or marked point process as an additive functional of the Markov process representing the network. The typical aim is to describe the behavior of the point process in terms of the characteristics of the Markov process. Immediate questions in this regard are: Is such a process Poisson (or marked Poisson)? Is a collection of these point processes multi-variate Poisson (or marked Poisson); and what are the dependencies among them?

We have found general easy-to-use criteria for establishing whether functionals of Markov processes are Poisson, compound Poisson or multi-variate Poisson. Although our applications were motivated by characterizing batch-service queues and batch movements in networks of

queues, our results apply to Markov processes in general. Furthermore, our approach has some extensions to semi-markov processes and other non-Markovian stationary processes.

### 3. International Telecommunications Networks

This work appeared in:

Serfozo, R.f. (1986). Equitable transit charges for multi-administration telecommunications networks. Queueing Systems 2, 83-92. (The abstract appears in Appendix A3.)

The International Telegraph and Telephone Consultative Committee, associated with the United Nations, meets every four years to set standards for international telecommunications. One issue they are addressing is the possibility of countries (i.e. separate telecommunications administrations) sharing communication links (satellite or cable circuits). Two major problems in designing an international network are: (1) determining the optimal numbers of circuits to have between countries and (2) determining a method of sharing the costs equitably. We worked on these problems at AT&T Bell Laboratories and continued to work on problem (2) at Georgia Tech, culminating in the publication above.

In this work, we focused on networks with alternate routing of calls via other countries' circuits, and sharing the costs by imposing transit charges on alternately-routed calls. For instance, if a call between the USA

and New Zealand is alternately routed through Australia, then the USA and New Zealand would split the revenue for the call and then pay Australia a transit charge for using its circuits. The issue is to determine an equitable transit charge. There are a number of situations where such transit charges for domestic as well as international telecommunications have been set "behind closed doors" with profits in mind. The difficulty we faced was to find a "scientific method" for doing this that was indeed equitable so the countries would accept it.

We were able to solve this problem by modeling an international network as a network of stochastic service systems, where the circuits are the servers. We obtained a formula for equitable transit charges by equating the hypothetical networks that would minimize the countries' costs (the individuals' preferred networks) to the most economically efficient network (the socially preferred network). This approach was feasible because of the mathematical structure of the cost functions. This approach might have applications in designing other systems involving cost sharing among several companies or individuals.

#### **4. Optimal Control of Networks of Queues**

A first draft of some of our work on this topic is the following: Menich, R. and R.F. Serfozo (1986). Optimality of shortest queue routing for dependent service stations. Technical report. (The abstract is in Appendix A4.)

We have been working on several problems in this area. This year we documented some preliminary work on optimal routing in networks of dependent service systems. For simple systems it is obvious that customers should be routed to the shortest queue to minimize congestion. We are attempting to identify when this policy is optimal for more complicated networks. Some of our results are in the reference above. We have obtained further results in the regard that will be documented next month.

We have also solved an analogous problem for a single server that can serve one of several separate flows of customers; the question being which flow should the server serve at any time? Again, for simple systems, it is obvious that the server should serve the flow with the longest queue. We have identified a large class of more complicated systems for which this policy is optimal. This problem has a similar mathematical structure as the one above.



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\*This is to be a chapter for the Operations Research Society of America Handbook on Stochastic Processes.

Poisson Functionals of Markov Processes  
and Queueing Networks

by

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ABSTRACT

We present conditions under which a point process of certain jump times of a Markov process is a Poisson process. One result is that if the Markov process is stationary and the compensator of the point process in reverse time has a constant intensity  $a$ , then the point process is Poisson with rate  $a$ . A classical example is that the output flow from a M/M/1 queueing system is Poisson. We also present similar Poisson characterizations of more general marked point process functionals of a Markov process. These results yield easy-to-use criteria for a collection of such processes to be multi-variate Poisson or marked Poisson with a specified dependence or independence. We give several applications to queueing systems, and indicate how our results extend to functionals of non-Markovian processes.

Keywords and phrases: Poisson process, multivariate compound Poisson process, functionals of Markov processes, queueing networks, time reversal.

This research was sponsored in part by Air Force Office of Scientific Research contracts S4-0367 and F49620 85 C 0144.

Queueing Systems 2 (1987) 83-92

## **EQUITABLE TRANSIT CHARGES FOR MULTI-ADMINISTRATION TELECOMMUNICATIONS NETWORKS \***

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(Revised 9 December 1986)

### **Abstract**

We study a multi-administration telecommunications network that is an abstraction of an international network. The nodes represent separate telecommunications administrations that are linked such that alternately-routed calls go through one tandem administration. The cost of the group of circuits between a pair of administrations is borne by them; and when a call between the pair is alternately routed through the tandem node, then the two administrations share the call revenue and pay transit fees to the tandem administration. The numbers of circuits between the administrations are selected to yield a least-cost network that provides a desired level of service, in terms of blocking probabilities, over an entire day. We address the problem of determining transit charges for the alternately-routed calls that are equitable for all of the administrations. Our approach is to derive such charges by equating the system-optimal circuit group sizes to certain hypothetical administration-optimal circuit group sizes. This approach may be of use in other system design problems involving cost sharing among several companies.

### **Keywords**

Telecommunications networks, queues, optimal design, equitable charges.

### **1. Introduction**

The accelerated growth of international teletraffic has prompted the world community to consider the formation of international networks with alternate routing of calls among countries. The International Telegraph and Telephone Consultative Committee (CCITT) has been considering standards for the design of such networks; see AT&T [2], Bell Northern Research [4], and Teleglobe [19].

\* This research was supported in part by Grant AFOSR 84-0367.

OPTIMALITY OF SHORTEST QUEUE ROUTING  
FOR DEPENDENT SERVICE STATIONS

by

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Abstract

We consider a system in which customers arrive singly or in batches to a group of service stations. A system controller observes the queues at the stations and routes each customer irrevocably to one of the stations for service. The controller wishes to minimize the infinite horizon expected cost of holding customers in the system. We show that it is optimal to route each arriving customer to the shortest queue in Markovian systems in which the systems may operate dependently and the arrival, service and holding cost rates are functions of the queue lengths. In particular, shortest queue routing is optimal when each station has  $s$  independent, identical servers (a  $M/M/s$  station). We establish our results by a Markov decision process argument.

Keywords: Queueing, service systems, Markov decision processes, stochastic control,  $M/M/s$  queues.

This research was supported in part by the Air Force Office of Scientific Research under contracts 84-0367 and F49620 85 C 0144.